

# Nonlinear Realization of N=2 Superconformal Symmetry and Brane Effective Actions

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**Abstract.** Due to the incompatibility of the nonlinear realization of superconformal symmetry and dilatation symmetry with the dilaton as the compensator field, in the present paper it shows an alternative mechanism of spontaneous breaking the N=2 superconformal symmetry to the N=0 case. By using the approach of nonlinear transformations it is found that it leads to a space-filling brane theory with Weyl scale  $W(1,3)$  symmetry. The dynamics of the resulting Weyl scale invariant brane, along with that of other Nambu-Goldstone fields, is derived in terms of the building blocks of the vierbein and the covariant derivative from the Maurer-Cartan one-forms. A general coupling of the matter fields localized on the brane world volume to these NG fields is also constructed.

## I. Introduction

In 1960s, being a powerful tool nonlinear realization method was used to study the low energy dynamics of the chiral symmetry [1], especially for the case when the full symmetry was spontaneously broken and the partners of the Goldstone bosons became massive and decoupled from the dynamics of the NG bosons. Later on, in Ref.[2] a general approach of nonlinear realization was given in terms of compact semisimple Lie groups, in which the full symmetry group was realized in terms of the nonlinear transformations of the NG fields which were promoted from the exponential parameters of the Coset representative elements, and these transformations were isomorphic to the left action of the general group elements on the Coset. Therefore, the transformations of the NG fields, together with that of the spectator fields which transform linearly under the unbroken subgroup  $H$ , give a complete representation of the full symmetry group. The resulting phenomenological Lagrangian, which is model independent, becomes an effective theory at energies far below the scale of spontaneous symmetry breaking. Nonlinear realization was also extended to include spacetime symmetries [3, 4], in which the spacetime translational generators and the broken symmetry generators transform independently under the stability group. Recently, extensive research work of nonlinear realization has been extended to describe the dynamics of the brane theories [5, 6, 7]. Consider a topological defect embedded in a target space. Its world volume then has the symmetries of the unbroken stability subgroup, whereas its long wave oscillations into the co-dimensional (super) space are described by the Nambu-Goldstone (Goldstino) modes associated with those broken symmetries [6, 7].

In this paper, we construct the nonlinear realization of spontaneously broken  $N=2$  superconformal symmetry by using Weyl scale invariant brane theories. As for the extended SUSY theory which includes more than one spinor supercharge, it is possible that it may arise from higher dimensional theories in some supersymmetric models or in some effective theories derived from higher dimensions via dimensional reductions. It is also well known that it plays an important role in understanding nonperturbative aspects of supersymmetric theories [8]. In the present context, it has been discussed that the  $N=2$  superconformal symmetry could be spontaneously broken by different approaches [9]. It may be partially broken down to the  $N=1$  Su-

per Poincare symmetry or to N=2 Super Poincare symmetry. Since supersymmetry must be broken for a realistic theory, in the present paper we consider the case when N=2 superconformal symmetry is totally broken down to N=0 supersymmetry.

It has been pointed out that the nonlinear realizations of supersymmetry and dilatation symmetry are not compatible when the dilaton is taken as the compensator field [10]. Therefore, in the present paper, one of its purposes is to show an alternative mechanism that describes the total supersymmetry breaking of the N=2 superconformal group. As it shows below, it would lead to a theory with Weyl scale invariant symmetry.

Consider a model, a space filling brane embedded in D=4 spacetime. Taking the static gauge, the spacetime coordinates  $x^\mu$  therefore line up with the variables  $\xi^\mu$  which parameterize the brane world volume, i.e.  $\xi^\mu = x^\mu$  (we take static gauge in what follows of this paper). The target superspace has the N=2 superconformal symmetry, and the embedded submanifold has unbroken Weyl  $W(1,3)$  symmetry, which is formed by the set  $\{P_\mu, D, M_{\mu\nu}\}$ . Then the low energy effective action of the system, which is scale invariant under the transformation of  $\xi^\mu \rightarrow e^d \xi^\mu$ , can be described by long wave oscillations of the brane into the superspace associated to the Grassmann coordinate directions  $\lambda, \bar{\lambda}$ , as well as the dynamics of the Nambu-Goldstone mode corresponding to broken generator  $A$  of the internal space.

On the other hand, the purpose of the paper is to explore more features of the well known *AdS/CFT* correspondence [11, 12]. In the spirit of the *AdS/CFT* correspondence, following the outline sketched by this paper one can find it paves a way to embed a probe brane into a  $AdS \times S$  background and realize the supersymmetric isometry of the target space. Associated with the brane, it is expected that there would be no destabilized terms due to the branes oscillations into the transverse spatial directions. We hope that would shed some light on the appealing aspects of the *AdS/CFT* correspondence in terms of the brane world scenarios.

The paper is organized as follows. First, in section II, we introduce the N=2 superconformal algebra, and construct the Coset structure in terms of the unbroken  $W(1,3)$  subgroup. The general infinitesimal transformations of the coordinates, as well as that of the  $Q_{\alpha i}$  type Goldstinos, are introduced through the action of the full group elements on the Coset representatives, whileas in section III the  $S^{\alpha i}$

type Goldstino spinors are proved to be superfluous and are eliminated by imposing covariant constraints. Also in that section, it follows that the vierbein and the covariant derivative of the NG field can be derived by means of the Maurer-Cartan one-forms. The effective scale invariant brane action is then constructed in terms of these building blocks. In section IV, the general coupling of the matter fields localized on the brane world volume to these NG modes is introduced. The infeasibility of taking the Lorentz group as the stability group for the spontaneous breaking of the full group is also pointed out, which is due to the fact that the nonlinear realization of the N=2 superconformal symmetry and the dilatation symmetry are not compatible when considering the dilaton field as the compensator field.

## II. Nonlinear Realization of the N=2 Superconformal Symmetry

The N=2 Superconformal algebra is isomorphic to the simple Lie superalgebra  $su(2, 2|2)$ , real form of  $sl(4|2)$ . Its algebra includes the conformal algebra:

$$\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\rho}M_{\mu\sigma} - \eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\sigma}M_{\mu\rho}) \\
[P_\mu, M_{\kappa\lambda}] &= i(\eta_{\mu\kappa}P_\lambda - \eta_{\mu\lambda}P_\kappa), [K_\mu, M_{\kappa\lambda}] = i(\eta_{\mu\kappa}K_\lambda - \eta_{\mu\lambda}K_\kappa) \\
[P_\mu, D] &= iP_\mu, [K_\mu, D] = -iK_\mu, [M_{\mu\nu}, D] = 0 \\
[P_\mu, K_\nu] &= 2i(\eta_{\mu\nu}D - M_{\mu\nu})
\end{aligned} \tag{1}$$

in which  $\eta^{\mu\nu} = (+, -, -, -, -)$ . It also has two different types of spinor charges  $Q_{\alpha i}$  and  $\bar{S}_i^{\dot{\alpha}}$ , and the commutation relations of these fermion-type charges with the conformal group generators and the internal group  $SU(2) \times U(1)_R$  generators have the form

$$\begin{aligned}
[Q_{\alpha i}, K_\mu] &= \sigma_{\mu\alpha\dot{\beta}}\bar{S}_i^{\dot{\beta}}, [\bar{S}_i^{\dot{\alpha}}, K_\mu] = 0, [\bar{S}_i^{\dot{\alpha}}, P_\mu] = \bar{\sigma}_\mu^{\dot{\alpha}\beta}Q_{\beta i} \\
[Q_{\alpha i}, D] &= \frac{1}{2}iQ_{\alpha i}, [Q_{\alpha i}, A] = \frac{1}{2}Q_{\alpha i}, [\bar{S}_i^{\dot{\alpha}}, D] = -\frac{1}{2}i\bar{S}_i^{\dot{\alpha}}, [S^{\alpha i}, A] = -\frac{1}{2}S^{\alpha i} \\
[Q_\alpha, M_{\mu\nu}] &= \frac{1}{2}(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta, [\bar{Q}^{\dot{\alpha}}, M_{\mu\nu}] = \frac{1}{2}(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}}\bar{Q}^{\dot{\beta}} \\
[Q_{\alpha i}, T_j^k] &= \delta_i^k Q_{\alpha j} - \frac{1}{2}\delta_j^k Q_{\alpha i}, [\bar{S}_i^{\dot{\alpha}}, T_j^k] = \delta_i^k \bar{S}_j^{\dot{\alpha}} - \frac{1}{2}\delta_j^k \bar{S}_i^{\dot{\alpha}}
\end{aligned} \tag{2}$$

When  $N > 1$ , there is no central charge, and the (anti)commutation relations among the fermion-type charges are

$$\{Q_{\alpha i}, \bar{Q}_{\dot{\alpha}}^j\} = 2\delta_i^j \sigma_{\alpha\dot{\alpha}}^\mu P_\mu, \{\bar{S}_i^{\dot{\alpha}}, S^{\beta j}\} = 2\delta_i^j \bar{\sigma}^{\mu\dot{\alpha}\beta} K_\mu$$

$$\begin{aligned} \{Q_{\alpha i}, Q_{\beta j}\} &= 0, \{Q_{\alpha i}, \bar{S}_j^{\dot{\beta}}\} = 0, \{\bar{S}_i^{\dot{\alpha}}, \bar{S}_j^{\dot{\beta}}\} = 0 \\ \{Q_{\alpha i}, S^{\beta j}\} &= \delta_i^j [(\sigma^{\mu\nu})_{\alpha}^{\beta} M_{\mu\nu} - 2iD\delta_{\alpha}^{\beta}] - 4\delta_{\alpha}^{\beta} (T_i^j + \frac{1}{2}\delta_i^j A); \end{aligned} \quad (3)$$

we take the notation  $\varepsilon_{\alpha\beta} = \varepsilon_{\dot{\alpha}\dot{\beta}} = -\varepsilon^{\alpha\beta} = -\varepsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ , and  $\sigma_{\mu\nu} = \frac{1}{2}i(\sigma_{\mu}\bar{\sigma}_{\nu} - \sigma_{\nu}\bar{\sigma}_{\mu})$ . Where  $A$  is the generator of  $U(1)_R$ , and the  $SU(2)$  generators  $T_i^j$  satisfy

$$[T_i^j, T_k^l] = \delta_i^l T_k^j - \delta_k^j T_i^l \quad (4)$$

where  $i, j = 1, 2$ . Consider the case when the N=2 superconformal symmetry is spontaneously broken down to the N=0 case. We choose the unbroken subgroup as

$$W(1, 3) \times SU(2) \quad (5)$$

in which  $W(1, 3)$  is the Weyl group, formed by the set  $\{P_{\mu}, D, M_{\mu\nu}\}$ [10]. Therefore, the group elements of the stability subgroup  $H$  are written as

$$h = e^{i(fD + m^{\mu\nu} M_{\mu\nu} + t_i^j T_i^j)} \quad (6)$$

which is spanned by the set of generators  $\{D, M_{\mu\nu}, T_i^j\}$ , and from Eqs.(1, 2) we can find these generators are automorphism of the broken generators  $Q, \bar{Q}, S, \bar{S}, K^{\mu}, A$  which are associated with the collective coordinates  $\lambda, \bar{\lambda}, \varsigma, \bar{\varsigma}, \phi^{\mu}$  and  $a$  respectively. Therefore, the Coset is shown to be

$$\Omega = G/H \quad (7)$$

In static gauge their representative elements can be parameterized as

$$\Omega = e^{ix^{\mu} P_{\mu}} e^{i[\lambda Q + \bar{\lambda} \bar{Q}]} e^{i(\varsigma S + \bar{\varsigma} \bar{S})} e^{i\phi^{\mu} K_{\mu}} e^{iaA} \quad (8)$$

The left action of the general infinitesimal elements of the full group  $G$

$$g = e^{i(a^{\mu} P_{\mu} + qQ + \bar{q}\bar{Q} + sS + \bar{s}\bar{S} + b^{\mu} K_{\mu} + rA + \rho^{\mu\nu} M_{\mu\nu} + dD + \varepsilon_j^i T_i^j)} \quad (9)$$

on the Coset representative elements of Eq. (8) can be uniquely decomposed as the product of the Coset  $\Omega'$  and the stability group elements of  $H$ , i.e.

$$g\Omega = \Omega' h \quad (10)$$

Explicitly, we have

$$\begin{aligned}
& e^{i(a^\mu P_\mu + qQ + \bar{q}\bar{Q} + sS + \bar{s}\bar{S} + b^\mu K_\mu + rA + \rho^{\mu\nu} M_{\mu\nu} + dD + \varepsilon_j^i T_i^j)} e^{ix^\mu P_\mu} e^{i[\lambda Q + \bar{\lambda}\bar{Q}]} e^{i(\varsigma S + \bar{\varsigma}\bar{S})} e^{i\phi^\mu K_\mu} e^{iaA} \\
& = e^{ix'^\mu P_\mu} e^{i[\lambda'Q + \bar{\lambda}'\bar{Q}]} e^{i(\varsigma' S + \bar{\varsigma}'\bar{S})} e^{i\phi'^\mu K_\mu} e^{ia'A} e^{i(fD + m^{\mu\nu} M_{\mu\nu} + t_j^i T_i^j)}
\end{aligned} \tag{11}$$

The infinitesimal transformation of the preferred fields can be derived up to the first order by considering the variation  $\delta g$  of the group elements  $g$  with

$$\delta g = i(a^\mu p_\mu + qQ + \bar{q}\bar{Q} + sS + \bar{s}\bar{S} + b^\mu K_\mu + rA + \rho^{\mu\nu} M_{\mu\nu} + dD + \varepsilon_j^i T_i^j) \tag{12}$$

Hence, Eq.(10) becomes

$$(1 + \delta g)\Omega = (\Omega + \delta\Omega)(1 + \delta h) \tag{13}$$

furthermore

$$\Omega^{-1}\delta g\Omega - \Omega^{-1}\delta\Omega = \delta h \tag{14}$$

Then it is found

$$\begin{aligned}
& \Omega^{-1}i(a^\mu p_\mu + qQ + \bar{q}\bar{Q} + sS + \bar{s}\bar{S} + b^\mu K_\mu + rA + \\
& \rho^{\mu\nu} M_{\mu\nu} + dD + \varepsilon_j^i T_i^j)\Omega - \Omega^{-1}\delta\Omega \\
& = i(fD + m^{\mu\nu} M_{\mu\nu} + t_j^i T_i^j)
\end{aligned} \tag{15}$$

Consider the pure shift induced by Poincare translation in the four dimensional spacetime, i.e., taking  $g = e^{ia^\mu P_\mu}$ , it is found

$$\delta x^\mu = a^\mu, \delta\lambda = \delta\varsigma = \delta\phi^\mu = \delta a = 0 \tag{16}$$

and

$$f = m^{\mu\nu} = t_j^i = 0. \tag{17}$$

Also, as for the pure shift  $g = e^{i(qQ + \bar{q}\bar{Q})}$  in the superspace, we have

$$\begin{aligned}
\delta x^\mu &= -i(\lambda^i \sigma^\mu \bar{q}_i - q^i \sigma^\mu \bar{\lambda}_i); \\
\delta\lambda^i &= q^i; \delta\bar{\lambda}_i = \bar{q}_i;
\end{aligned} \tag{18}$$

as well as  $\delta\varsigma = \delta\phi^\mu = \delta a = f = m^{\mu\nu} = t_j^i = 0$ . Here  $\delta\lambda^i$  etc are the total variation of the fields, i.e.  $\delta\lambda^i = \lambda'(x') - \lambda(x)$ , and the intrinsic variation of the Goldstino fields  $\lambda^i$  is given by

$$\delta_{in}\lambda^i = \lambda'(x') - \lambda(x) = \delta\lambda^i - (\lambda(x') - \lambda(x))$$

$$= q^i + i(\lambda^j \sigma^\mu \bar{q}_j - q^j \sigma^\mu \bar{\lambda}_j) \partial_\mu \lambda^i(x) \quad (19)$$

which is just the Akulov-Volkov nonlinear transformation of the Goldstino fields [14, 15] for the extended Supersymmetries. Besides, by comparing the coefficients of the  $Q_i$  and  $P_\mu$  from both sides of Eq.(15) the general nonlinear transformations of the coordinates and the associated Goldstino fields can be derived as following (the Goldstino fields  $\varsigma_i$  corresponding to  $S^i$  are superfluous and can be written as functions of  $\bar{\lambda}_i(x)$ , see Eq.(32')):

$$\begin{aligned} \delta x^\mu &= a^\mu - i2(\lambda^i \sigma^\mu \bar{q}_i - q^i \sigma^\mu \bar{\lambda}_i) + i(\lambda^i \sigma^\mu d\bar{\lambda}_i - d\lambda^i \sigma^\mu \bar{\lambda}_i) + r\lambda^i \sigma^\mu \bar{\lambda}_i \\ &\quad - \frac{1}{3!}i(2\lambda^i \sigma^\mu B_{4i} - 2A_4^i \sigma^\mu \bar{\lambda}_i) - \frac{1}{3!}i(2\lambda^i \sigma^\mu B_{5i} - 2A_5^i \sigma^\mu \bar{\lambda}_i) \\ &\quad - 2\rho^{\mu\nu} x_\nu + (x^{\mu'} b^{\nu'} - \frac{1}{2}\rho^{\mu'\nu'}) (\lambda^i \sigma^\mu \bar{\lambda}_i \bar{\sigma}_{\mu'\nu'} - \lambda^i \sigma_{\mu'\nu'} \sigma^\mu \bar{\lambda}_i) - \\ &\quad \frac{1}{4!} \cdot 2i(\lambda^i \sigma^\mu B_{6i} - A_6^i \sigma^\mu \bar{\lambda}_i) + 2x^\nu b_\nu x^\mu - x^\nu x_\nu b^\mu + d \cdot x^\mu \\ &\quad + \varepsilon_k^j (2\lambda^k \sigma^\mu \bar{\lambda}_j - \lambda^i \sigma^\mu \delta_j^k \bar{\lambda}_i) + 2\lambda^i \sigma^\nu x_\nu s_i \sigma^\mu - 2x^\nu \bar{s}^i \bar{\sigma}_\nu \sigma^\mu \bar{\lambda}_i \end{aligned} \quad (20)$$

and

$$\begin{aligned} \delta \lambda^i &= q^i - i\frac{1}{2}r\lambda^i + ix^\mu \bar{s}^i \bar{\sigma}_\mu + \frac{1}{2}A_5^i + \frac{1}{2}A_4^i - \frac{1}{2}i\rho^{\mu\nu} \lambda^i \sigma_{\mu\nu} + \frac{1}{3!}A_6^i + \frac{1}{2}d \cdot \lambda^i \\ &\quad + x^\mu b_\mu \lambda^i + ix^\mu b^\nu \lambda^i \sigma_{\mu\nu} - i\varepsilon_k^j (\lambda^k \delta_j^i - \frac{1}{2}\delta_j^k \lambda^i); \\ \delta \bar{\lambda}_i &= \bar{q}_i + i\frac{1}{2}r\bar{\lambda}_i + ix^\mu s_i \sigma^\mu + \frac{1}{2}B_{5i} + \frac{1}{2}B_{4i} - \frac{1}{2}i\rho^{\mu\nu} \bar{\lambda}_i \bar{\sigma}_{\mu\nu} + \frac{1}{3!}B_{6i} + \frac{1}{2}d \cdot \bar{\lambda}_i \\ &\quad + x^\mu b_\mu \bar{\lambda}_i + ix^\mu b^\nu \bar{\lambda}_i \bar{\sigma}_{\mu\nu} + i\varepsilon_k^j (\bar{\lambda}_j \delta_i^k - \frac{1}{2}\delta_j^k \bar{\lambda}_i) \end{aligned} \quad (21)$$

where the inhomogeneous terms play important roles in the nonlinear transformation and signal the spontaneous breaking of the extended supersymmetries. (see Appendix for definitions of the  $A$  s and  $B$  s in these equations)

### III. The Effective Action

The effective action can be derived by using the Maurer-Cartan one-forms which are expanded with respect to the  $su(2, 2|2)$  generators:

$$\begin{aligned} \Omega^{-1} d\Omega &= i(\omega^a P_a + \omega_Q^k Q_k + \bar{\omega}_{\bar{Q}k} \bar{Q}^k + \omega_{S^k} S^k + \bar{\omega}_{\bar{S}^k} \bar{S}^k + \omega_k^a K_a + \omega_A A \\ &\quad + \omega_D D + \omega_T j^i T_i^j + \omega_M^{ab} M_{ab}) \end{aligned} \quad (22)$$

Explicitly, they have the form

$$\begin{aligned}
i\omega^a &= id x^a + \lambda^i \sigma^\mu d\bar{\lambda}_i - d\lambda^i \sigma^\mu \bar{\lambda}_i; \\
i\omega_Q^k &= dx^\mu \bar{\zeta}^k \bar{\sigma}_\mu e^{\frac{1}{2}ia} + id\lambda^k e^{\frac{1}{2}ia} - i(\lambda^i \sigma^\mu d\bar{\lambda}_i - d\lambda^i \sigma^\mu \bar{\lambda}_i) \bar{\zeta}^k \bar{\sigma}_\mu e^{\frac{1}{2}ia}; \\
i\bar{\omega}_{\bar{Q}k} &= dx^\mu \zeta_k \sigma_\mu e^{-\frac{1}{2}ia} + id\bar{\lambda}_k e^{-\frac{1}{2}ia} - i(\lambda^i \sigma^\mu d\bar{\lambda}_i - d\lambda^i \sigma^\mu \bar{\lambda}_i) \zeta_k \sigma_\mu e^{-\frac{1}{2}ia}; \\
i\omega_{S^k} &= (i\phi^\mu dx^\nu \zeta_k \sigma_\nu \bar{\sigma}_\mu + \frac{1}{3!} A_{1k}|_{a^\mu=id x^\mu}) e^{-i\frac{a}{2}} + [\frac{1}{2} A_{2k}|_{q_i=id\lambda_i} + \frac{1}{2} A_3 A_{3k}|_{\bar{q}_i=id\bar{\lambda}_i} + \\
&\quad \frac{1}{3!} A_{1k}|_{a^\mu=\lambda\sigma^\mu d\bar{\lambda}-d\lambda\sigma^\mu \bar{\lambda}} + i\phi^\mu (id\bar{\lambda}_k - i(\lambda^i \sigma^\nu d\bar{\lambda}_i - d\lambda^i \sigma^\nu \bar{\lambda}_i) \zeta_k \sigma_\nu) \bar{\sigma}_\mu] e^{-i\frac{a}{2}} + id\zeta_k e^{-i\frac{a}{2}} \\
i\omega_{\bar{S}^k} &= (i\phi^\mu dx^\nu \bar{\zeta}^k \bar{\sigma}_\nu \sigma_\mu + \frac{1}{3!} B_1^k|_{a^\mu=id x^\mu}) e^{i\frac{a}{2}} + [\frac{1}{2} B_2^k|_{q^i=id\lambda_i} + \frac{1}{2} B_3^k|_{\bar{q}^i=id\lambda_i} + \\
&\quad \frac{1}{3!} B_1^k|_{a^\mu=\lambda\sigma^\mu d\bar{\lambda}-d\lambda\sigma^\mu \bar{\lambda}} - \phi^\mu (d\bar{\lambda}^k - (\lambda^i \sigma^\nu d\bar{\lambda}_i - d\lambda^i \sigma^\nu \bar{\lambda}_i) \bar{\zeta}^k \bar{\sigma}_\nu) \sigma_\mu] e^{i\frac{a}{2}} + id\bar{\zeta}^k e^{i\frac{a}{2}} \\
i\omega_k^a &= -i\phi^{\mu'} \phi_{\mu'} dx^a - (\bar{\zeta}^i \bar{\sigma}_\mu \varsigma_i + \bar{\zeta}^i \varsigma_i \sigma_\mu) dx^\mu \phi^a + \frac{1}{4!} 2i (B_1^k|_{a^\mu=id x^\mu} \bar{\sigma}^a \varsigma_k - \\
&\quad \bar{\zeta}^k \bar{\sigma}^a A_{1k}|_{a^\mu=id x^\mu}) - i\bar{\zeta}^i (\bar{\sigma}_{\mu'} \sigma^{a\mu'} \varsigma_i - \bar{\sigma}^{a\mu'} \bar{\zeta}_i \sigma_{\mu'}) dx^{\mu'} \phi_\nu + X^a \\
&\quad - d\bar{\zeta}^i \bar{\sigma}^a \varsigma_i + \bar{\zeta}^i \bar{\sigma}^a d\varsigma_i + id\phi^a; \\
i\omega_A &= ida + 2d\lambda^i \varsigma_i - 2\bar{\zeta}^i d\bar{\lambda}_i - \bar{\zeta}^i \bar{\sigma}_\mu (\lambda^j \sigma^\mu d\bar{\lambda}_j - d\lambda^j \sigma^\mu \bar{\lambda}_j) \varsigma_i \\
&\quad + \bar{\zeta}^i \varsigma_i \sigma_\mu (\lambda^j \sigma^\mu d\bar{\lambda}_j - d\lambda^j \sigma^\mu \bar{\lambda}_j) + i(-\bar{\zeta}^i \bar{\sigma}_\mu \varsigma_i + \bar{\zeta}^i \varsigma_i \sigma_\mu) dx^\mu; \\
i\omega_D &= -2i\phi^\mu dx_\mu + (\bar{\zeta}^i \bar{\sigma}_\mu \varsigma_i + \bar{\zeta}^i \varsigma_i \sigma_\mu) dx^\mu + 2id\lambda^i \varsigma_i + 2i\bar{\zeta}^i d\bar{\lambda}_i \\
&\quad - i\bar{\zeta}^i \varsigma_i \sigma_\mu (\lambda^j \sigma^\mu d\bar{\lambda}_j - d\lambda^j \sigma^\mu \bar{\lambda}_j) - 2\varphi^\mu (\lambda^j \sigma_\mu d\bar{\lambda}_j - d\lambda^j \sigma_\mu \bar{\lambda}_j) \\
&\quad - i\bar{\zeta}^i \bar{\sigma}_\mu (\lambda^j \sigma^\mu d\bar{\lambda}_j - d\lambda^j \sigma^\mu \bar{\lambda}_j) \varsigma_i; \\
i\omega_{T^j_i} &= 4d\lambda^j \varsigma_i - 4\bar{\zeta}^j d\bar{\lambda}_i - 2\bar{\zeta}^j \bar{\sigma}_\mu (\lambda^k \sigma^\mu d\bar{\lambda}_k - d\lambda^k \sigma^\mu \bar{\lambda}_k) \varsigma_i \\
&\quad + 2\bar{\zeta}^j \varsigma_i \sigma_\mu (\lambda^k \sigma^\mu d\bar{\lambda}_k - d\lambda^k \sigma^\mu \bar{\lambda}_k) + 2i(-\bar{\zeta}^i \bar{\sigma}_\mu \varsigma_j + \bar{\zeta}^i \varsigma_j \sigma_\mu) dx^\mu; \\
i\omega_M^{ab} &= dx^{\mu'} [2i\phi^b \delta_{\mu'}^a + \frac{i}{2} (\bar{\zeta}^i \bar{\sigma}_{\mu'} \sigma^{ab} \varsigma_i - \bar{\zeta}^i \bar{\sigma}^{ab} \varsigma_i \sigma_{\mu'})] - d\lambda^i \sigma^{ab} \varsigma_i + \bar{\zeta}^i \bar{\sigma}^{ab} d\bar{\lambda}_i \\
&\quad + \frac{1}{2} \bar{\zeta}^i \bar{\sigma}_\mu (\lambda^j \sigma^\mu d\bar{\lambda}_j - d\lambda^j \sigma^\mu \bar{\lambda}_j) \sigma^{ab} \varsigma_i - \frac{1}{2} \bar{\zeta}^i \bar{\sigma}^{ab} \varsigma_i \sigma_\mu (\lambda^j \sigma^\mu d\bar{\lambda}_j - d\lambda^j \sigma^\mu \bar{\lambda}_j) \\
&\quad + 2\phi^b (\lambda^i \sigma^a d\bar{\lambda}_i - d\lambda^i \sigma^a \bar{\lambda}_i); \tag{23}
\end{aligned}$$

where the NG fields are the Weyl spinors  $\lambda^i$ ,  $\varsigma_i$  and the axion  $a$ , corresponding to the broken generators  $Q_i$ ,  $S^i$  and  $A$  respectively. And  $\phi^\mu$  is the independent collective coordinate in the Coset space. It is not treated as the dynamical field [16,17]. In addition, not all the NG fields are independent fields. Note from Eq.(2) that

$$[\bar{S}_i^{\dot{\alpha}}, P_\mu] = \bar{\sigma}_\mu^{\dot{\alpha}\beta} Q_{\beta i} \tag{24}$$



Or alternatively, we have

$$[\bar{s}_\alpha^i \bar{S}_i^{\dot{\alpha}} + s_i^\alpha S_\alpha^i, P_\mu] = \bar{s}_\alpha^i \bar{\sigma}_\mu^{\dot{\alpha}\beta} Q_{\beta i} + s_i^\alpha \sigma_{\mu\alpha\dot{\alpha}} \bar{Q}^{\dot{\alpha}i} \quad (25)$$

Consider the commutator of NG fields  $\lambda^i$  of the broken Supercharges  $Q_i$  with Eq.(25),

$$[[\bar{s}_\alpha^i \bar{S}_i^{\dot{\alpha}} + s_i^\alpha S_\alpha^i, P_\mu], \lambda^{\gamma i}] = [\bar{s}_\alpha^i \bar{\sigma}_\mu^{\dot{\alpha}\beta} Q_{\beta i} + s_i^\alpha \sigma_{\mu\alpha\dot{\alpha}} \bar{Q}^{\dot{\alpha}i}, \lambda^{\gamma i}] \quad (26)$$

The VEV of the right-hand side can be found from the infinitesimal supersymmetric transformation of  $\lambda^i$ :

$$\langle |[\bar{s}_\alpha^i \bar{\sigma}_\mu^{\dot{\alpha}\beta} Q_{\beta i} + s_i^\alpha \sigma_{\mu\alpha\dot{\alpha}} \bar{Q}^{\dot{\alpha}i}, \lambda^{\alpha i}]| \rangle \sim \bar{s}_\alpha^i \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \quad (27)$$

Hence, applying the Jacobin identity to the left-hand side of Eq.(26) yields

$$\langle |[\bar{s}_\alpha^i \bar{S}_i^{\dot{\alpha}} + s_i^\alpha S_\alpha^i, i\partial_\mu \lambda^{\gamma i}]| \rangle \sim -\bar{s}_\alpha^i \bar{\sigma}_\mu^{\dot{\alpha}\beta} \neq 0 \quad (28)$$

On the other hand, the NB fields  $\bar{\zeta}^i$  of the broken Supercharge  $\bar{S}_i$  have the infinitesimal supersymmetric transformation properties

$$\langle |[\bar{s}_\alpha^i \bar{S}_i^{\dot{\alpha}} + s_i^\alpha S_\alpha^i, \bar{\zeta}_\gamma^i]| \rangle \sim \bar{s}_\gamma^i \neq 0 \quad (29)$$

Therefore, from Eqs.(28,29) we can conclude that

$$i\partial_\mu \lambda^{\gamma i} \sim -\bar{\zeta}_\alpha^i \bar{\sigma}_\mu^{\dot{\alpha}\beta} \quad (30)$$

Or it can be re-written as

$$\bar{\zeta}_\gamma^i \sim -\frac{1}{4} i\partial_\mu \lambda^{\alpha i} \sigma_{\alpha\gamma}^\mu \quad (31)$$

Consequently,  $\bar{\zeta}^i$  are not independent NG fields and can be written as a function of the fields  $\lambda^i$ . In order to eliminate them from the effective action, consider the covariant constraints on the one-forms  $\omega_Q^i$ , i.e.,  $\omega_Q^i = 0$ . Hence, it is found

$$\bar{\zeta}_\gamma^i = -\frac{1}{4} i e_a^{-1\mu} \partial_\mu \lambda^{\alpha i} \sigma_{\alpha\gamma}^a \quad (32)$$

where  $e_a^{-1\mu}$  is the inverse of the vierbein (see Eq.(33) for definition). This is just the inverse Higgs mechanism [4]. Similarly, from  $\bar{\omega}_{\bar{Q}} = 0$ , it amounts to

$$\zeta_i^\alpha = -\frac{1}{4} i e_a^{-1\mu} \partial_\mu \bar{\lambda}_{\gamma i} \bar{\sigma}^{a\gamma\alpha} \quad (32')$$

Therefore, the real Nambu-Goldstone (Goldstino) degrees of freedom are these fields  $\lambda^i$  and  $a$ . And the effective action of these modes can be constructed by using the building blocks of verbein and covariant derivatives from the one-forms of Eq.(23).

Consider the covariant coordinate one-forms  $\omega^a$ , which can be decomposed with respect to the brane world volume coordinate differential one-forms  $d\xi^\mu$  as  $\omega^a = d\xi^\mu e_\mu^a$ . The verbein is therefore given by

$$e_\mu^a = \frac{\partial x^a}{\partial \xi^\mu} - i(\lambda^i \sigma^a \frac{\partial \bar{\lambda}_i}{\partial \xi^\mu} - \frac{\partial \lambda^i}{\partial \xi^\mu} \sigma^a \bar{\lambda}_i) \quad (33)$$

where  $x^a = (x^0, x^1, x^2, x^3)$ , and it becomes

$$e_\mu^a = \delta_\mu^a - i(\lambda^i \sigma^a \partial_\mu \bar{\lambda}_i - \partial_\mu \lambda^i \sigma^a \bar{\lambda}_i) \quad (33')$$

in static gauge  $\xi^\mu = x^\mu$ . Hence, the local Lorentz invariant interval becomes

$$ds^2 = g_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{ab} \omega^a \omega^b \quad (34)$$

where the metric  $g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b$ . We use Latin letters  $a, b$  etc to represent the local tangent coordinate indices, and Greek letters  $\mu, \nu$  etc for these of the general coordinates in what follows. We also take the static gauge unless explicitly indicated otherwise. In order to construct the covariant derivative of the field  $a$ , since  $dx^\mu = \omega^a e_a^{-1\mu}$ , the one-forms  $\omega_A$  can be re-written as  $\omega_A = \omega^a D_a a$ , in which

$$\begin{aligned} D_a a = e_a^{-1\mu} & (\partial_\mu a - i2\partial_\mu \lambda^i \varsigma_i + i2\bar{\varsigma}^i \partial_\mu \bar{\lambda}_i + i\bar{\varsigma}^i \bar{\sigma}_{\mu'} (\lambda^j \sigma^{\mu'} \partial_\mu \bar{\lambda}_j - \partial_\mu \lambda^j \sigma^{\mu'} \bar{\lambda}_j) \varsigma_i \\ & - i\bar{\varsigma}^i \varsigma_i \sigma_{\mu'} (\lambda^j \sigma^{\mu'} \partial_\mu \bar{\lambda}_j - \partial_\mu \lambda^j \sigma^{\mu'} \bar{\lambda}_j) + (-\bar{\varsigma}^i \bar{\sigma}_\mu \varsigma_i + \bar{\varsigma}^i \varsigma_i \sigma_\mu)) \end{aligned} \quad (35)$$

On the other hand, under the action of  $g$  of Eq.(10), the Maurer-Cartan one-forms transform according to

$$\Omega'^{-1} d\Omega' = h(\Omega^{-1} d\Omega) h^{-1} + h d h^{-1} \quad (36)$$

It thus follows that the dilatation transformation properties of the verbein and covariant derivative in the local Lorentz reference frame are found to be

$$\begin{aligned} \omega^a & \rightarrow e^d \omega^a \\ e_\mu^a & \rightarrow e^d e_\mu^a \\ D_a a & \rightarrow e^{-d} D_a a \end{aligned} \quad (37)$$

i.e., their scale dimensions are 1,  $-2$ , and  $-1$  respectively. In addition, considering the transformation of Eq.(37), they are found to be  $SU(2)$  singlets.

Introduce an auxiliary (intrinsic) metric  $G_{\mu\nu}$  whose scale dimension is 2 as induced by the scale transformation of  $\xi \rightarrow e^d \xi^\mu$  on the brane world volume. The local Lorentz invariant interval  $ds^2$  has the form

$$ds^2 = G_{\mu\nu} d\xi^\mu d\xi^\nu \quad (38)$$

which has scale dimension 2 as a result of scale transformation  $ds^2 \rightarrow e^{2d} ds^2$ .

Then the effective scale invariant action of the brane world volume is given by

$$\begin{aligned} I_0 &= -\frac{f_s^2}{2} \int d^4 \xi \sqrt{|\det G|} \left[ \frac{1}{4} G^{\mu\nu} \eta_{ab} e_\mu^a e_\nu^b \right]^2 \\ &= -\frac{f_s^2}{2} \int d^4 \xi \sqrt{|\det G|} \left[ \frac{1}{4} G^{\mu\nu} \eta_{ab} \left( \frac{\partial x^a}{\partial \xi^\mu} - i(\lambda^i \sigma^a \frac{\partial \bar{\lambda}_i}{\partial \xi^\mu} - \frac{\partial \lambda^i}{\partial \xi^\mu} \sigma^a \bar{\lambda}_i) \right) \right. \\ &\quad \cdot \left. \left( \frac{\partial x^b}{\partial \xi^\nu} - i(\lambda^i \sigma^b \frac{\partial \bar{\lambda}_i}{\partial \xi^\nu} - \frac{\partial \lambda^i}{\partial \xi^\nu} \sigma^b \bar{\lambda}_i) \right) \right]^2 \end{aligned} \quad (39)$$

in which  $\det G = \det G_{\mu\nu}$  and the tensor  $G^{\mu\nu}$  with scale dimension is  $-2$  is the inverse of  $G_{\mu\nu}$ . The part inside the square brackets has a scale dimension  $-2$ . It can be concluded that Eq.(39) is Weyl scale invariant under the transformation of  $\xi^\mu \rightarrow e^d \xi^\mu$ . Obviously, when the spinors  $\lambda^i$  are set to zero, it reduces to the Weyl scale invariant bosonic action [18]:

$$I_B = -\frac{f_s^2}{2} \int d^4 \xi \sqrt{|\det G|} \left[ \frac{1}{4} G^{\mu\nu} \eta_{ab} \frac{\partial x^a}{\partial \xi^\mu} \frac{\partial x^b}{\partial \xi^\nu} \right]^2 \quad (40)$$

In static gauge, Eq.(39) becomes

$$\begin{aligned} I_0 &= -\frac{f_s^2}{2} \int d^4 x \sqrt{|\det G|} \left[ \frac{1}{4} G^{\mu\nu} \eta_{ab} (\delta_\mu^a - i(\lambda^i \sigma^a \partial_\mu \bar{\lambda}_i - \partial_\mu \lambda^i \sigma^a \bar{\lambda}_i)) \right. \\ &\quad \cdot (\delta_\nu^b - i(\lambda^i \sigma^b \partial_\nu \bar{\lambda}_i - \partial_\nu \lambda^i \sigma^b \bar{\lambda}_i)) \left. \right]^2 \end{aligned} \quad (41)$$

Besides, the spinors  $\lambda^i$  transform as a  $SU(2)$  doublet, i.e.

$$\lambda'^i = e^{(it_{k'}^{j'} T_{j'}^{k'})^i_j} \lambda^j \quad (42)$$

where the  $SU(2)$  operators  $T_{j'}^{i'}$ , with properties  $(T_j^i)^\dagger = T_i^j$ , have the matrix representation  $(T_{j'}^{i'})^i_j = \delta_j^i \delta_{j'}^{i'} - \frac{1}{2} \delta_{j'}^{i'} \delta_j^i$ . Besides, the spinors  $\bar{\lambda}_i$  can be found transforming as  $SU(2)$  covariant vectors:

$$\bar{\lambda}'_i = e^{-(it_{k'}^{j'} T_{j'}^{k'})^j_i} \bar{\lambda}_j \quad (43)$$

It is thus obvious that  $i(\lambda^i \sigma^b \partial_\nu \bar{\lambda}_i - \partial_\nu \lambda^i \sigma^b \bar{\lambda}_i)$  is SU(2) invariant. As a result, the action of Eq.(41) is both scale and SU(2) invariant.

When considering the dynamics of the Goldstinos  $\lambda^i$  alone, the auxiliary field  $G_{\mu\nu}$  can be eliminated from Eq.(41) after applying its equation of motion

$$G_{\mu\nu} = \Lambda e_\mu^a e_\nu^b \eta_{ab} \quad (44)$$

where  $\Lambda$  is an arbitrary constant. Then after plugging Eq.(44) into Eq.(41), it reduces to

$$I'_0 = -\frac{f_s^2}{2} \int d^4x \det e_\mu^a \quad (45)$$

in which the coefficient  $f_s$  is related to the SUSY broken scale. It is just the Akulov-Volkov action for the case of the extended supersymmetries. Similarly, applying the equation of motion of  $G_{\mu\nu}$  to the Weyl scale invariant bosonic action of Eq.(40) will lead to the normal Nambu-Goto action

$$I'_B = -\frac{f_s^2}{2} \int d^4\xi \sqrt{\left| \det(\eta_{ab} \frac{\partial x^a}{\partial \xi^\mu} \frac{\partial x^b}{\partial \xi^\nu}) \right|} \quad (46)$$

As for the effective scale invariant action of the SU(2) singlet, i.e. the NG field  $a$ , it can be constructed by using the covariant derivative of Eq.(35):

$$I_1 = T_A \int d^4x \sqrt{|\det G|} F(G^{\mu\nu}, e_\mu^a) \eta^{ab} D_a a D_b a \quad (47)$$

where the compensator function  $F(G^{\mu\nu}, e_\mu^a)$  has the scale dimension  $-2$ , and the coefficient and  $T_A$  is related to breaking scale of  $A$  symmetry. Hence, the complete effective action of the fields  $\lambda^i$  and  $a$  is given by

$$\begin{aligned} I &= I_0 + I_1 \\ &= \int d^4x T_A \sqrt{|\det G|} F(G^{\mu\nu}, e_\mu^a) \eta^{ab} D_a a D_b a - \\ &\quad \frac{f_s^2}{2} \sqrt{|\det G|} \left[ \frac{1}{4} G^{\mu\nu} (\delta_{\mu\nu} - i(\lambda^i \sigma_\mu \partial_\nu \bar{\lambda}_i - \partial_\nu \lambda^i \sigma_\mu \bar{\lambda}_i) \right. \\ &\quad \left. - i(\lambda^i \sigma_\nu \partial_\mu \bar{\lambda}_i - \partial_\mu \lambda^i \sigma_\nu \bar{\lambda}_i) - (\lambda^i \sigma_b \partial_\mu \bar{\lambda}_i - \partial_\mu \lambda^i \sigma_b \bar{\lambda}_i)(\lambda^i \sigma^b \partial_\nu \bar{\lambda}_i - \partial_\nu \lambda^i \sigma^b \bar{\lambda}_i) \right]^2 \end{aligned} \quad (48)$$

#### IV. Coupling to Matter Fields

Consider the presence of matter fields localized on the brane world volume. They actually behave as the spectator fields, transforming covariantly under the unbroken subgroup  $H$ . Their coupling to those NG fields can be obtained by introducing covariant derivatives of the matter fields. Consider matter fields  $\Phi_A$ , in which  $A$  represents any internal or Lorentz index. It has a  $N$ -dimensional representation of the group  $SU(2)$ , i.e.

$$\Phi_A \rightarrow \Phi'_A = e^{it_j^i (T_i^j)_A^B} \Phi_B \quad (49)$$

likewise, its Hermitian conjugate transforms as

$$\Phi^A \rightarrow \Phi'^A = e^{-it_j^i (T_i^j)_B^A} \Phi^B \quad (50)$$

And under the full unbroken subgroup  $H$ , the matter field would transform according to

$$\Phi \rightarrow \Phi' = e^{i(fD + m^{\mu\nu} M_{\mu\nu} + t_j^i T_i^j)} \Phi \quad (51)$$

Accordingly, its covariant derivative is given by

$$D_\mu \Phi = (\partial_\mu + i \frac{1}{2} \omega_\mu^{ab} \sum_{ab} + i d_\Phi \omega_{D\mu} + i \omega_{T\mu j}^i T_i^j) \Phi \quad (52)$$

Where  $\sum_{ab}$  is the representation of the operators  $M_{ab}$ , and  $d_\Phi$  is the scale dimension of the matter field. The connections  $\omega_\mu^{ab}$ ,  $\omega_{D\mu}$  and  $\omega_{T\mu i}^j$  are given by the one-forms  $\omega_M^{ab} = dx^\mu \omega_\mu^{ab}$ ,  $\omega_D = dx^\mu \omega_{D\mu}$  and  $\omega_{T\mu i}^j = dx^\mu \omega_{T\mu i}^j$  respectively. They transform inhomogeneously according to Eq.(36). Explicitly, the transformation of the spin connection  $\omega_\mu^{ab}$

$$\begin{aligned} \omega_\mu^{ab} = & 2\phi^b \delta_\mu^a + \frac{1}{2} (\bar{\varsigma}^i \bar{\sigma}_\mu \sigma^{ab} \varsigma_i - \bar{\varsigma}^i \bar{\sigma}^{ab} \varsigma_i \sigma_\mu) + i(\partial_\mu \lambda^i \sigma^{ab} \varsigma_i - \bar{\varsigma}^i \bar{\sigma}^{ab} \partial_\mu \bar{\lambda}_i) \\ & - i \frac{1}{2} \bar{\varsigma}^i \bar{\sigma}_{\mu'} (\lambda^j \sigma^{\mu'} \partial_\mu \bar{\lambda}_j - \partial_\mu \lambda^j \sigma^{\mu'} \bar{\lambda}_j) \sigma^{ab} \varsigma_i + \\ & i \frac{1}{2} \bar{\varsigma}^i \bar{\sigma}^{ab} \varsigma_i \sigma_{\mu'} (\lambda^j \sigma^{\mu'} \partial_\mu \bar{\lambda}_j - \partial_\mu \lambda^j \sigma^{\mu'} \bar{\lambda}_j) - i 2\phi^b (\lambda^i \sigma^a \partial_\mu \bar{\lambda}_i - \partial_\mu \lambda^i \sigma^a \bar{\lambda}_i) \end{aligned} \quad (53)$$

is found to have the synthesis form of the general coordinate transformation of Eq.(20) and the inhomogeneous transformation under  $h$  according to Eq.(36):

$$\omega_{\mu'}^{cd} = \frac{dx^\mu}{dx^{\mu'}} (\omega_\mu^{a'b'} \Lambda_{a'}^c \Lambda_{b'}^d - \partial_\mu m^{cd}) \quad (54)$$

where the  $\Lambda^c_d$  matrix is defined by the ordinary Lorentz transformation,  $x^{a'} = \Lambda^{a'}_b x^b$ ; and the inhomogeneous term can be derived from that of Eq.(36) as following

$$\begin{aligned} h d h^{-1} &= e^{i(fD + m^{\mu\nu} M_{\mu\nu} + t_j^i T_i^j)} d e^{-i(fD + m^{\mu\nu} M_{\mu\nu} + t_j^i T_i^j)} \\ &= -i d m^{ab} M_{ab} - i d f D - i d t_j^i T_i^j \end{aligned} \quad (55)$$

which has been expanded up to the leading order of infinitesimal transformation of  $g$  in Eq.(9).

Similarly, it follows from Eq.(23) that the connections  $\omega_D = dx^\mu \omega_{D\mu}$  has the form

$$\begin{aligned} \omega_{D\mu} &= -2\phi_\mu - i(\bar{\varsigma}^i \bar{\sigma}_\mu \varsigma_i + \bar{\varsigma}^i \varsigma_i \sigma_\mu) + 2\partial_\mu \lambda^i \varsigma_i + 2\bar{\varsigma}^i \partial_\mu \bar{\lambda}_i \\ &\quad - \bar{\varsigma}^i \varsigma_i \sigma_\nu (\lambda^j \sigma^\nu \partial_\mu \bar{\lambda}_j - \partial_\mu \lambda^j \sigma^\nu \bar{\lambda}_j) + i 2\phi^\nu (\lambda^j \sigma_\nu \partial_\mu \bar{\lambda}_j - \partial_\mu \lambda^j \sigma_\nu \bar{\lambda}_j) \\ &\quad - \bar{\varsigma}^i \bar{\sigma}_\nu (\lambda^j \sigma^\nu \partial_\mu \bar{\lambda}_j - \partial_\mu \lambda^j \sigma^\nu \bar{\lambda}_j) \varsigma_i \end{aligned} \quad (56)$$

in which the non dynamical field  $\phi^\mu$  can be eliminated by imposing the covariant constraint on the covariant curl  $C_{\mu\nu}^a$  [17], i.e.

$$\begin{aligned} C_{\mu\nu}^a &= 0 = D_\mu e_\nu^a - D_\nu e_\mu^a \\ &= \partial_\mu e_\nu^a + i \frac{1}{2} \omega_\mu^{a'b'} \left( \sum_{a'b'} \right)_b^a e_\nu^b + i \omega_{D\mu} e_\nu^a - \partial_\nu e_\mu^a \\ &\quad - i \frac{1}{2} \omega_\nu^{a'b'} \left( \sum_{a'b'} \right)_b^a e_\mu^b - i \omega_{D\nu} e_\mu^a \end{aligned} \quad (57)$$

Consider the scalar field  $\phi$ , which has trivial representation of  $M_{ab}$ . Therefore, the covariant derivative becomes  $D_\mu \phi = (\partial_\mu - i \omega_{D\mu} + i \omega_{T\mu j}^i T_i^j) \phi$ . Then the scale invariant action of the scalar field is shown to be

$$\begin{aligned} I_\Phi &= \int d^4x \det e g^{\mu\nu} D_\mu \phi D_\nu \phi \\ &= \int d^4x \det e \eta^{ab} e_a^{-1\mu} e_b^{-1\nu} (\partial_\mu - i \omega_{D\mu} + i \omega_{T\mu j}^i T_i^j) \phi \\ &\quad \cdot (\partial_\nu + i \omega_{D\nu} - i \omega_{T\nu j}^i T_i^j) \phi \end{aligned} \quad (58)$$

and the scalar field has scale dimension  $-1$ . Considering Eq.(49) and (50), it is also obviously  $SU(2)$  invariant.

Similarly, consider the fermion fields  $\psi$ , which give a nontrivial representation of  $M_{ab}$ , i.e.  $\sum_{ab} = \frac{1}{2} i(\gamma_a \gamma_b - \gamma_b \gamma_a)$ . Its covariant derivative is then found to be

$D_\mu\psi = (\partial_\mu + i\frac{1}{2}\omega_\mu^{ab}\sum_{ab} - i\frac{3}{2}\omega_{D\mu} + i\omega_{T\mu j}^i T_i^j)\psi$ . Thus its scale and SU(2) invariant action is given by

$$\begin{aligned} I_\psi &= \int d^4x \det e \bar{\psi} i \gamma^\mu D_\mu \psi \\ &= \int d^4x \det e \bar{\psi} i \gamma^a e_a^{-1\mu} (\partial_\mu + i\frac{1}{2}\omega_\mu^{ab}\sum_{ab} - i\frac{3}{2}\omega_{D\mu} + i\omega_{T\mu j}^i T_i^j) \psi \end{aligned} \quad (59)$$

in which the fermion field has the scale dimension  $-\frac{3}{2}$ . Therefore the total action of the matter fields coupling to these NG fields can be described by

$$\begin{aligned} I_M &= I_\Phi + I_\Psi \\ &= \int d^4x \det e (\eta^{ab} e_a^{-1\mu} e_b^{-1\nu} (\partial_\mu - i\omega_{D\mu} + i\omega_{T\mu j}^i T_i^j) \phi \\ &\quad \cdot (\partial_\nu + i\omega_{D\nu} - i\omega_{T\nu j}^i T_i^j) \phi \\ &\quad + \bar{\psi} i \gamma^a e_a^{-1\mu} (\partial_\mu + i\frac{1}{2}\omega_\mu^{ab}\sum_{ab} - i\frac{3}{2}\omega_{D\mu} + i\omega_{T\mu j}^i T_i^j) \psi) \end{aligned} \quad (60)$$

As a result, on the brane world volume the unbroken symmetries  $H$  are realized linearly on the localized spectator fields, whileas the broken symmetries are realized nonlinearly through the Nambu-Goldstone(Goldstino) modes  $a$  and  $\lambda^i$ . On the other hand, the full symmetry can also be directly realized on the field itself, such as the standard realization [15, 19]. One can start from the linear transformation of the matter fields under the stability group  $H$ , then promote it to describe the full symmetry by taking the parameters of  $h$  as these induced form the left action of  $G$  on the Coset representatives through Eq.(10). Hence the transformation of the fields can realize the full symmetry group  $G$ , but nonlinearly.

The Weyl scale invariant brane dynamics has also been extensively studied in [18,20-23]. The action of Eq.(48), being a low energy effective theory, describes the long wave oscillations of the brane into the Grassmann coordinates of the superspace along with the effective dynamics of the localized Nambu-Goldstone mode  $a$  corresponding to the broken  $A$  symmetry. Consequently, by using the approach of nonlinear realization, the total action of Eq.(48) gives us an effective theory describing the spontaneously breaking of a larger symmetry group, i.e. the N=2 superconformal symmetry. In fact, the Weyl scale invariant brane action Eq.(41) is an extension of its bosonic counterpart action Eq.(40) to include the fermionic

sectors as a result of the full supersymmetry breaking. Therefore, the fields localized on the brane world volume would be described by the Weyl scale symmetry theory [13, 24-27].

Besides, the internal symmetry  $SU(2) \times U(1)_R$  of the extended N=2 Superconformal symmetry can also be broken down to  $U(1)_R$  [28]. Hence, corresponding to generators  $T_i^j$ , there would be three NG scalar fields present in the effective action along with other possible  $SU(2)$  broken terms. The existence of these NG particles would give rise to long range forces in nature, which may affect astrophysical considerations such as contributing new mechanisms for energy loss from stars [19].

In addition, it is noteworthy that if one takes the Lorentz group as the stability group instead, the dilatation symmetry is also spontaneously broken. Therefore, it will result an additional NG field, the dilaton  $\sigma$ , whose intrinsic transformation yields

$$\sigma' \sim d + \sigma \quad (61)$$

Therefore, the scale invariant effective action of the Goldstino fields becomes

$$I_0 = -T \int d^4x e^{-4\sigma} \det e \quad (62)$$

where the dilaton is introduced as the compensator field. From Eq.(33'), it can be concluded that there is a constant term in  $\det e$  which shifts the VEV and signals the spontaneous breaking of the SUSY. When considering together with the compensator field  $e^{-4\sigma}$ , the VEV can be determined by estimating the value of  $\langle |e^{-4\sigma}| \rangle$ , which becomes minimum when  $\langle |\sigma| \rangle$  goes to  $\infty$ . Therefore, due to the unbound of the VEV of the dilaton field it follows that  $\sigma$  can not be a NG particle. It thus indicates the incompatibility of the nonlinear realization of the SUSY and the dilatation symmetry when taking dilaton as the compensator field [10, 29]. It is interesting to note that if one takes the Lorentz group as the stability group and works on the superspace parameterized by the Coset space, it will lead to the supersymmetric theory (superbrane) as discussed in [5, 30]. However, this is not our case in the present context.

Besides, due to the absence of the dilaton in the effective action (41), according to the  $AdS/CFT$  correspondence, one may expect it is feasible to embed a probe brane in a supersymmetric  $AdS \times S$  space. And following the outline of the present paper,



the supersymmetric isometry group of the background space can be realized through the dynamics of the brane but with no destabilized terms resulting from oscillations in the transverse spatial directions. That would be instructive to explore more aspects about  $AdS/CFT$  correspondence. Further work about this correspondence is being investigated and it would be of interest to the theory of brane world scenarios as well.

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## Appendix

The  $A$ s and  $B$ s in Eqs.(20,21,23) are defined as following:

(i)

$$e^{-i(\varsigma S + \bar{\varsigma} \bar{S})} a^\mu P_\mu e^{i(\varsigma S + \bar{\varsigma} \bar{S})} = \dots + \frac{1}{3!} (A_{1k} S^k + B_1^k \bar{S}_k) + \dots \quad (A.1)$$

where

$$\begin{aligned} A_{1k} = & -i(\bar{\varsigma}^i \bar{\sigma}_{\mu'} a^{\mu'} \sigma^{\mu\nu} \varsigma_i - \bar{\varsigma}^i \bar{\sigma}^{\mu\nu} \varsigma_i \sigma_{\mu'} a^{\mu'}) \frac{1}{2} \varsigma_k \sigma_{\mu\nu} \\ & + 4i(-\bar{\varsigma}^i \bar{\sigma}_\mu a^\mu \varsigma_j + \bar{\varsigma}^i \varsigma_j \sigma_\mu a^\mu) \varsigma_i \delta_k^j + 2i\bar{\varsigma}^i \bar{\sigma}_\mu a^\mu \varsigma_i \varsigma_k \end{aligned} \quad (A.2)$$

$$\begin{aligned} B_1^k = & -i(\bar{\varsigma}^i \bar{\sigma}_{\mu'} a^{\mu'} \sigma^{\mu\nu} \varsigma_i - \bar{\varsigma}^i \bar{\sigma}^{\mu\nu} \varsigma_i \sigma_{\mu'} a^{\mu'}) \frac{1}{2} \bar{\varsigma}^k \bar{\sigma}_{\mu\nu} \\ & - 4i(-\bar{\varsigma}^i \bar{\sigma}_\mu a^\mu \varsigma_j + \bar{\varsigma}^i \varsigma_j \sigma_\mu a^\mu) \bar{\varsigma}^j \delta_i^k + 2i\bar{\varsigma}^i \varsigma_i \sigma_\mu a^\mu \bar{\varsigma}^k \end{aligned} \quad (A.3)$$

(ii)

$$e^{-i(\varsigma S + \bar{\varsigma} \bar{S})} q Q e^{i(\varsigma S + \bar{\varsigma} \bar{S})} = \dots + \frac{1}{2} (A_{2k} S^k + B_2^k \bar{S}_k) + \dots \quad (A.4)$$

where

$$A_{2k} = \frac{1}{2} q^i \sigma^{\mu\nu} \varsigma_i \varsigma_k \sigma_{\mu\nu} + 4q^i \varsigma_j (\varsigma_i \delta_k^j - \frac{1}{2} \delta_i^j \varsigma_k) \quad (A.5)$$

$$B_2^k = \frac{1}{2} q^i \sigma^{\mu\nu} \varsigma_i \bar{\varsigma}^k \bar{\sigma}_{\mu\nu} - 4q^i \varsigma_j \bar{\varsigma}^j \delta_i^k \quad (A.6)$$

(iii)

$$e^{-i(\varsigma S + \bar{\varsigma} \bar{S})} \bar{q} \bar{Q} e^{i(\varsigma S + \bar{\varsigma} \bar{S})} = \dots + \frac{1}{2} (A_{3k} S^k + B_3^k \bar{S}_k) + \dots \quad (A.7)$$

where

$$A_{3k} = -\frac{1}{2} \bar{\varsigma}^i \bar{\sigma}^{\mu\nu} \bar{q}_i \varsigma_k \sigma_{\mu\nu} - 4\bar{\varsigma}^j \bar{q}_i \varsigma_j \delta_k^i \quad (A.8)$$

$$B_3^k = -\frac{1}{2} \bar{\varsigma}^i \bar{\sigma}^{\mu\nu} \bar{q}_i \bar{\varsigma}^k \bar{\sigma}_{\mu\nu} + 4\bar{\varsigma}^j \bar{q}_i (\bar{\varsigma}^i \delta_j^k - \frac{1}{2} \delta_j^i \bar{\varsigma}^k) \quad (A.9)$$

(iv)

$$e^{-i(\lambda Q + \bar{\lambda} \bar{Q})} s S e^{i(\lambda Q + \bar{\lambda} \bar{Q})} = \dots + \frac{1}{2} (A_4^k Q_k + B_{4k} \bar{Q}^k) + \dots \quad (A.10)$$

where

$$A_4^k = -\lambda^i \sigma^{\mu\nu} s_i \frac{1}{2} \lambda^k \sigma_{\mu\nu} + 4\lambda^i s_j (\lambda^j \delta_i^k - \frac{1}{2} \delta_i^j \lambda^k) \quad (A.11)$$

$$B_{4k} = -\lambda^i \sigma^{\mu\nu} s_i \frac{1}{2} \bar{\lambda}_k \bar{\sigma}_{\mu\nu} - 4\lambda^i s_j \bar{\lambda}_i \delta_k^j \quad (A.12)$$

(v)

$$e^{-i(\lambda Q + \bar{\lambda} \bar{Q})} \bar{s} \bar{S} e^{i(\lambda Q + \bar{\lambda} \bar{Q})} = \dots + \frac{1}{2} (A_5^k Q_k + B_{5k} \bar{Q}^k) + \dots \quad (A.13)$$

where

$$A_5^k = \bar{s}^i \bar{\sigma}^{\mu\nu} \bar{\lambda}_i \frac{1}{2} \lambda^k \sigma_{\mu\nu} - 4\bar{s}^i \bar{\lambda}_j \lambda^j \delta_i^k \quad (A.14)$$

$$B_{5k} = \bar{s}^i \bar{\sigma}^{\mu\nu} \bar{\lambda}_i \frac{1}{2} \bar{\lambda}_k \bar{\sigma}_{\mu\nu} + 4\bar{s}^i \bar{\lambda}_j (\bar{\lambda}_i \delta_k^j - \frac{1}{2} \delta_i^j \bar{\lambda}_k) \quad (A.15)$$

(vi)

$$e^{-i(\lambda Q + \bar{\lambda} \bar{Q})} b^\mu K_\mu e^{i(\lambda Q + \bar{\lambda} \bar{Q})} = \dots + \frac{1}{3!} (A_6^k Q_k + B_{6k} \bar{Q}^k) + \dots \quad (A.16)$$

where

$$A_6^k = -i \left[ \frac{1}{2} (-\lambda^i \sigma^{\mu\nu} b^{\alpha'} \bar{\lambda}_i \bar{\sigma}_{\alpha'} + b^{\mu'} \lambda^i \sigma_{\mu'} \bar{\sigma}^{\mu\nu} \bar{\lambda}_i) \lambda^k \sigma_{\mu\nu} - 2b^\mu \lambda^i \bar{\lambda}_i \bar{\sigma}_\mu \lambda^k + \right. \\ \left. 4b^\mu (\lambda^i \bar{\lambda}_j \bar{\sigma}_\mu - \lambda^i \sigma_\mu \bar{\lambda}_j) \lambda^j \delta_i^k \right] \quad (A.17)$$

$$B_{6k} = -i \left[ \frac{1}{2} (-\lambda^i \sigma^{\mu\nu} b^{\alpha'} \bar{\lambda}_i \bar{\sigma}_{\alpha'} + b^{\mu'} \lambda^i \sigma_{\mu'} \bar{\sigma}^{\mu\nu} \bar{\lambda}_i) \bar{\lambda}_k \bar{\sigma}_{\mu\nu} - 2b^\mu \lambda^i \sigma_\mu \bar{\lambda}_i \bar{\lambda}_k - \right. \\ \left. 4b^\mu (\lambda^i \bar{\lambda}_j \bar{\sigma}_\mu - \lambda^i \sigma_\mu \bar{\lambda}_j) \bar{\lambda}_i \delta_k^j \right] \quad (A.18)$$

(vii) In Eq.(23),  $X^\mu$  is defined as following

$$X^\mu = 2[-d\lambda^i \sigma^{\mu'\mu} \varsigma_i + \varsigma^i \bar{\sigma}^{\mu'\mu} d\bar{\lambda}_i + \frac{1}{2} (\bar{\varsigma}^i \bar{\sigma}_\nu a^\nu \sigma^{\mu'\mu} \varsigma_i - \bar{\varsigma}^i \bar{\sigma}^{\mu'\mu} \varsigma_i \sigma_\nu a^\nu)] \phi_{\mu'} \\ - (2id\lambda^i \varsigma_i + 2i\bar{\varsigma}^i d\lambda_i - i\varsigma^i \bar{\sigma}_{\mu'} a^{\mu'} \varsigma_i - i\bar{\varsigma}^i \varsigma_i \sigma_{\mu'} a^{\mu'}) \phi^\mu + \frac{1}{3!} 2i (B_2^k \bar{\sigma}^\mu \varsigma_k - \bar{\varsigma}^k \bar{\sigma}^\mu A_{2k}) + \\ + \frac{1}{3!} 2i (B_3^k \bar{\sigma}^\mu \varsigma_k - \bar{\varsigma}^k \bar{\sigma}^\mu A_{3k}) + \frac{1}{4!} 2i (B_1^k \bar{\sigma}^\mu \varsigma_k - \bar{\varsigma}^k \bar{\sigma}^\mu A_{1k}) - \phi^{\mu'} \phi_{\mu'} a^\mu \quad (A.19)$$

where  $a^\mu = \lambda \sigma^\mu d\bar{\lambda} - d\lambda \sigma^\mu \bar{\lambda}$ .

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